Email: ammarali@wrec.uoqasim.edu.iq

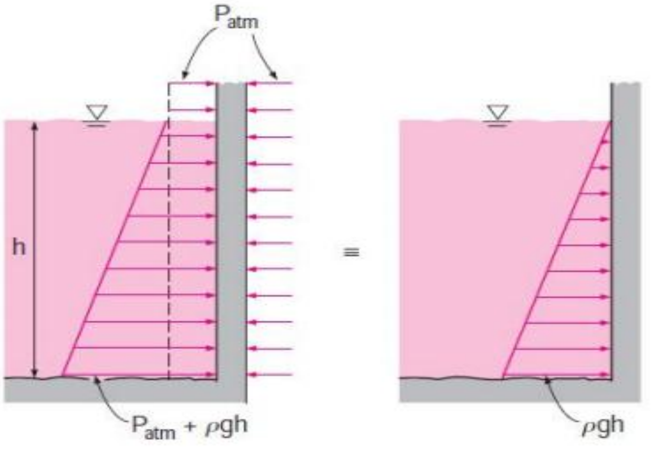
Hydrostatic Forces on Submerged Plane Surfaces

Hydrostatic forces mean forces exerted by fluid at rest.

- A plate exposed to a liquid, such as a gate valve in a dam, the wall of a liquid storage tank, or the hull of a ship at rest, is subjected to fluid pressure distributed over its surface.
- Hydrostatic forces form a system of parallel forces,
- We need to determine the magnitude of the force;
- and its point of application,

Centre of Pressure

Note: When analyzing hydrostatic forces on submerged surfaces; the atmospheric pressure can be subtracted for simplicity when it acts on both sides of the structure.



(a) P_{atm} considered (b) P_{atm} subtracted

Consider the plane surface shown in the figure below. The total area is made up of many elemental areas. The force on each elemental area is always normal to the surface but, in general, each force is of different magnitude as the pressure usually varies.

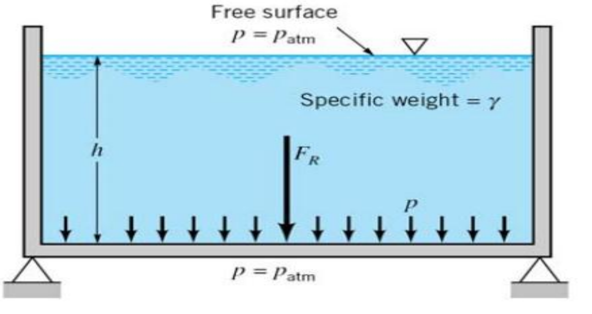
Forces on Horizontal Submerged surface:

For a horizontal plane submerged in a liquid (or a plane experiencing uniform pressure over its surface), the pressure, p, will be equal at all points of the surface. Thus the resultant force will be given by

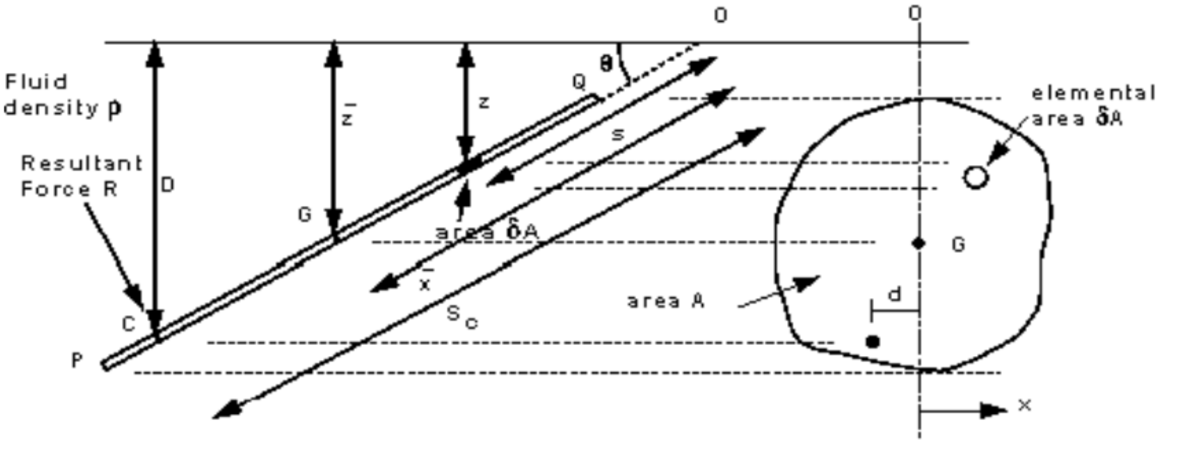
$P = \gamma h$: Uniform on the entire plane.

∴ Resultant force, $F_R = P.A = \gamma.A.h$

(A: the bottom area of container)



Forces on Inclined Submerged surface:



This plane surface is totally submerged in a liquid of density ρ and inclined at an angle of ϑ to the horizontal. Taking pressure as zero at the surface and measuring down from the surface, the pressure on an element γA , submerged distance z, is given by

$$P = \rho g Z$$

And therefore, the force on the element is

$$F = \gamma Z \delta A$$

The resultant force can be found by summing all of these forces:

$$F_R = \gamma \sum z\delta A$$

Where, $\sum z\delta A$ known as **1st Moment of Area** which is equal to AZ.

$\sum z\delta A = AZ =$ **1st Moment of Area about the line of free surface.**

Where A is the area of the plane and Z is the depth (distance from the free surface) to the centroid, G.

Steps to solve the exerted bodies problems:

1- First, we have the main equation to find the resultant forces:

$$F_R = \gamma . A . h_c$$

2- Location of the force:

$$y_R = \frac{I_{x,c}}{y_c . A} + y_c$$

3- h_c, for the vertical surface is the distance from the surface to the centre, for example if the depth 2.5, then the h_c =1.25.

4- h_c, for inclined surface, will be

$h_c = distance\ from\ the\ surface\ of\ the\ fluid\ to\ the\ up\ edge\ of\ the\ submerged$

$surface + length\ of\ the\ surface/2\ \ * sin\theta$

5- The **Area** term in the resultant force equation can be calculated directly, depend on the shape of submerged surface, for example, if the gate is square, the area will be (Width * Length) as so on.

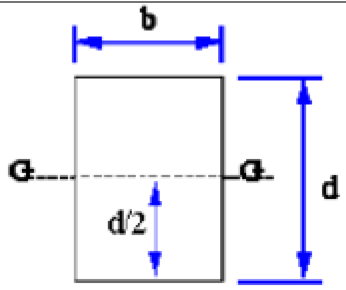
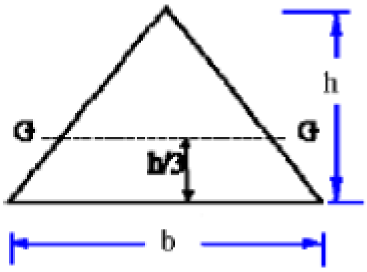
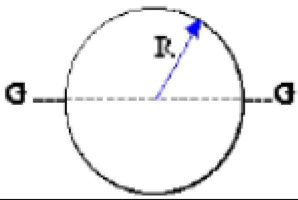
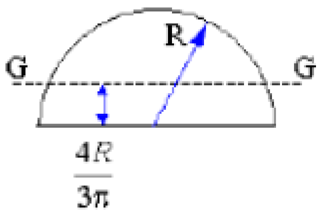
6- Now we find the distance where the force F acts from the hinge:

$$y_R = \frac{I_{x,c}}{y_c . A} + y_c$$

$$y_c = \frac{h_c}{sin\theta}\ , for\ inclined\ surface.$$

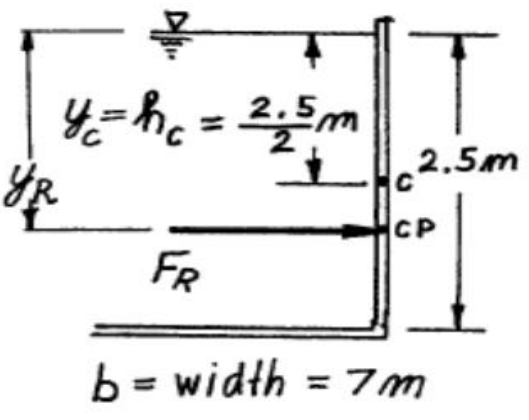
$$y_c = h_c\ , for\ vertical\ surface.$$

7- $I_{x,c}$, can be calculated based on the shape of the submerged body wich can be shown for common bodies:

Shape	Dimensions	Area	$I_{x,c}$, 2nd moment of area
Rectangle		bd	$\frac{bd^3}{12}$
Triangular		$\frac{bh}{2}$	$\frac{bh^3}{36}$
Circle		πR^2	$\frac{\pi R^4}{4}$
Semi-Circle		$\frac{\pi R^2}{2}$	$0.11R^4$

Example 1: A swimming pool is 18m long and 7m wide. Determine the magnitude and the location of the resultant force of the water on the vertical end of the pool where the depth is 2.5m.

Solution:



The force is

$$F_R = \gamma . A . h_c$$

$$F_R = 1000 * 9.81 * 2.5 * 7 * \frac{2.5}{2} = 214.6\ kN$$

The location of force:

$$y_R = \frac{I_{x,c}}{y_c . A} + y_c$$

$$y_c = h_c, for\ vertical\ surface \rightarrow\ y_c = \frac{2.5}{2} = 1.25\ m$$

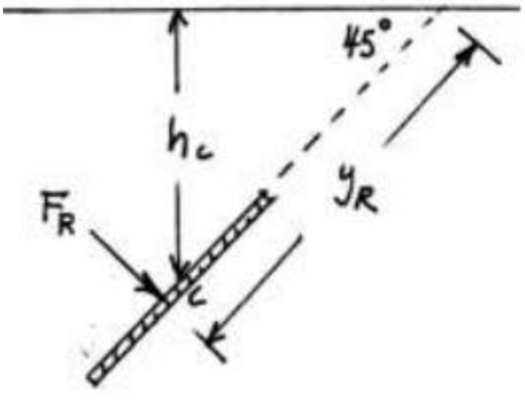
The second moment of area can be calculated from the table for the rectangular:

$$I_{x,c} = \frac{bd^3}{12} \rightarrow\ I_{x,c} = \frac{7 * 2.5^3}{12} = 9.115\ m^4$$

$$y_R = \frac{9.115}{1.25 * 7 * 2.5} + 1.25 = 1.66\ m$$

Example 2: A square 3m*3m gate is located in 45° sloping side of a dam. Some measurements indicate that the resultant force of the water on the gate is 500 kN.

1- Determine the pressure at the bottom of the gate.
2- Show where this force acts.



Solution:

1- The force is

$$F_R = \gamma . A . h_c$$

$$500 * 1000 = 1000 * 9.81 * 3 * 3 * h_c \rightarrow\ h_c = 5.66\ m$$

$$P = \gamma * \left(h_c + \frac{3}{2} * sin45\right) \rightarrow P = 65.9\ kN$$

2- The location of force:

$$y_R = \frac{I_{x,c}}{y_c.A} + y_c$$

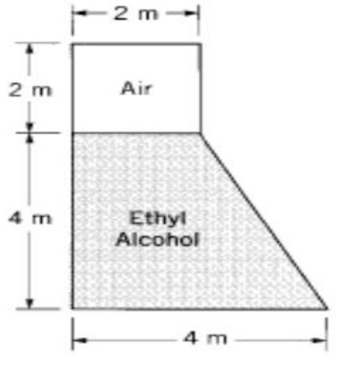
$$y_c = \frac{h_c}{sin\theta} \to y_c = \frac{5.66}{sin45} = 8\, m$$

The second moment of area can be calculated from the table for the rectangular:

$$I_{x,c} = \frac{bd^3}{12} \to I_{x,c} = \frac{3 * 3^3}{12} = 6.75\, m^4$$

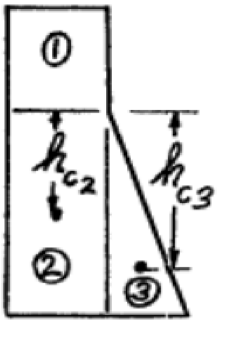
$$y_R = \frac{6.75}{8 * 3 * 3} + 8 = 8.09375\, m$$

Example 3: the vertical cross section of a 7m long closed storage tank is shown below. The tank contains ethyl alcohol and air pressure is 40 kpa. Determine the magnitude of the resultant fluid force acting on one end of the tank.



Solution:

Break the figure to three parts as shown below:



1- For part 1:

$$F_1 = \gamma.A.h_c \to F_R = P * A \to F_R = 40 * 2 * 2 = 160\, kN$$

2- For part 2:

$$F_R = P_{air} * A_2 + \gamma_{ethyl} * h_{c2} * A$$

$$F_2 = 40 * 4 * 2 + 7.74 * \left(\frac{4}{2}\right) * 4 * 2 = 444\ kN$$

3- For part 3:

$$F_R = P_{air} * A_2 + \gamma_{ethyl} * h_{c2} * A$$

$$F_3 = 40 * \frac{1}{2} * 4 * 2 + 7.74 * \left(\frac{2}{3} * 4\right) * \frac{1}{2} * 4 * 2 = 243\ kN$$

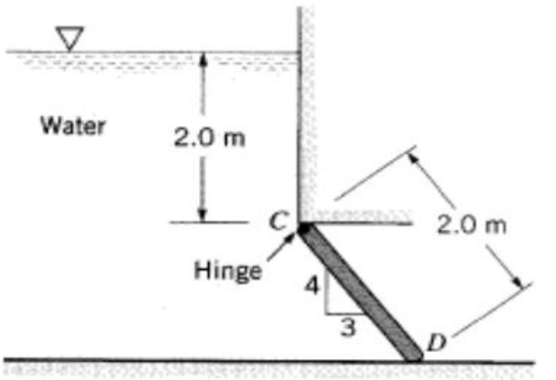
Area of part 3:

$$A = \frac{1}{2} * bottom * height\ ,\quad h_c = \frac{2}{3} * height$$

Now, the resultant force can be evaluated by:

$$F_R = F_1 + F_2 + F_3 = 160 + 444 + 243 = 847\ kN$$

Example 4: the rectangular gate CD shown in the below figure is 1.8m wide and 2m long. Assuming the material of the gate to be homogeneous and neglecting friction at the hinge C, determine the resultant and the location of this force of the gate necessary to keep it shut until the water level rises to 2m above the hinge.



Solution:

To start solving this, we need to find the angle first:

$$\theta = \tan^{-1}(\frac{4}{3}) = 53.13°$$

$$F_R = \gamma . A . h_c \rightarrow F_R = 1000 * 9.81 * 1.8 * 2 * (2 + \frac{2}{2} * \sin(53.13) = 98.88\ kN$$

Now, the location can be found by:

$$y_R = \frac{I_{x,c}}{y_c . A} + y_c$$

Where,

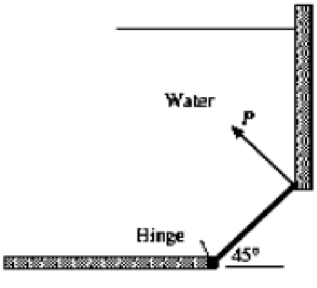
$$y_c = \frac{h_c}{sin53.13} = \frac{2 + sin53.13}{sin53.13} = 3.5m$$

And from table the second moment of area can be calculated by:

$$I_{x,c} = \frac{bd^3}{12} = \frac{1.8 * 2^3}{12} = 1.2\ m^4$$

$$y_R = \frac{1.2}{3.5 * 1.8 * 2} + 3.5 = 3.595m$$

Example 5: A 60-cm square gate (a = 0.6) has its top edge 12 m below the water surface. It is on a 45° angle and its bottom edge is hinged as shown in figure below. What force P is needed to just open the gate?



Solution: the force can be calculated by:

$$F_R = \gamma . A . h_c \rightarrow F_R = 1000 * 9.81 * 0.6 * 0.6 * (12 + \frac{0.6}{2} * \sin(45) = 53.1283\ kN$$

Now we find the distance d where the force F acts from the hinge:

$$y_R = \frac{I_{x,c}}{y_c . A} + y_c$$

Where,

$$y_c = \frac{h_c}{sin45} = \frac{12 + 0.3 * sin45}{sin45} = 17.27m$$

And from table the second moment of area can be calculated by:

$$I_{x,c} = \frac{bd^3}{12} = \frac{0.6 * 0.6^3}{12} = 0.0108\ m^4$$

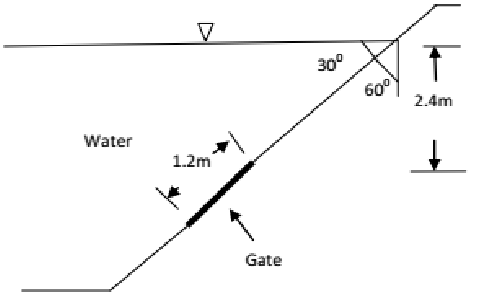
$$y_R = \frac{0.0108}{17.27 * 0.6 * 0.6} + 17.27 = 17.272m$$

$$d = 0.3 - (y_R - y_c) = 0.3 - (17.272 - 17.27) = 0.298m$$

The force P can be calculated:

$$P = d * \frac{F_R}{a} = 21.04kN$$

Example 6: An inclined rectangular gate (1.5m wide) contains water on one side. Determine the total resultant force acting on the gate and the location.



Solution: the force can be calculated by:

$$F_R = \gamma . A . h_c \rightarrow F_R = 1000 * 9.81 * 1.5 * 1.2 * (2.4 + \frac{1.2}{2} * \sin(30) = 47.6766\ kN$$

Now we find the distance d where the force F acts from the hinge:

$$y_R = \frac{I_{x,c}}{y_c . A} + y_c$$

Where,

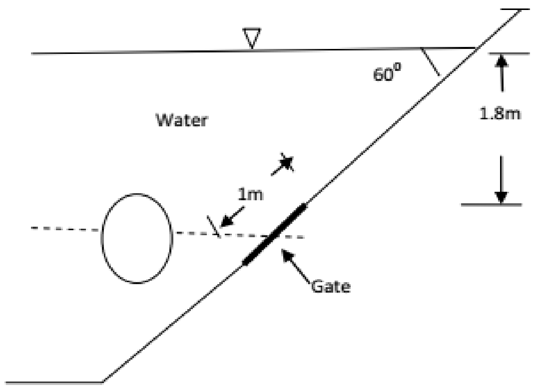
$$y_c = \frac{h_c}{sin30} = \frac{2.4 + \frac{1.2}{2} * sin30}{sin30} = 5.4m$$

And from table the second moment of area can be calculated by:

$$I_{x,c} = \frac{bd^3}{12} = \frac{1.5 * 1.2^3}{12} = 0.216\ m^4$$

$$y_R = \frac{0.216}{5.4 * 1.5 * 1.2} + 5.4 = 5.422m$$

Example 7: An inclined circular with water on one side is shown in the fig. Determine the total resultant force acting on the gate and the location.



Solution:

Area of the circular gate, $A = \frac{\pi}{4} * d^2 \rightarrow A = 0.785\ m^2$

the force can be calculated by:

$$F_R = \gamma . A . h_c \rightarrow F_R = 1000 * 9.81 * 0.785 * (1.8 + \frac{1}{2} * \sin(60)) = 17.2\ kN$$

Now we find the distance where the force F acts:

$$y_R = \frac{I_{x,c}}{y_c . A} + y_c$$

Where,

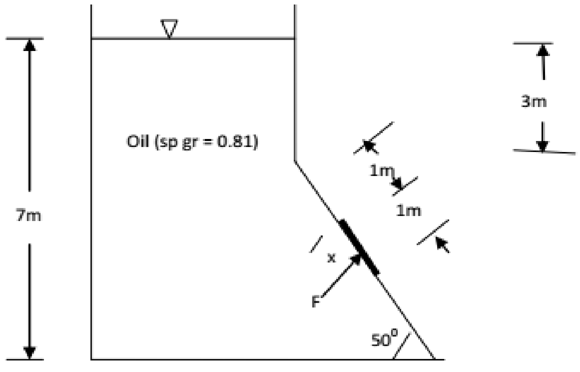
$$y_c = \frac{h_c}{sin\theta} = \frac{1.8 + \frac{1}{2} * sin60}{sin60} = 2.578m$$

And from table the second moment of area can be calculated by:

$$I_{x,c} = \frac{\pi R^4}{4} = \frac{\pi * 0.5^4}{4} = 0.049m^4$$

$$y_R = \frac{0.049}{2.578 * 0.785} + 2.578 = 2.6m$$

Example 8: Gate AB in the figure below is 1m long and 0.7m wide. Calculate force F on the gate and position X of c.p. (specific gravity of oil is 0.81)



Solution:

Area of the rectangular gate, $A = length * width \rightarrow A = 0.7\ m^2$

The density of oil can be calculated by:

$$\rho_{oil} = 0.81 * 1000 = 810\frac{kg}{m^3}$$

And h_c can be evaluated as below:

$$h_c = 3 + 1 * sin50 + \frac{1}{2} * sin50 = 4.15\ m$$

the force can be calculated by:

$$F_R = \gamma. A. h_c \rightarrow F_R = 810 * 9.81 * 0.7 * 4.15 = 23.0834\ kN$$

Now we find the distance where the force F acts:

$$y_R = \frac{I_{x,c}}{y_c. A} + y_c$$

Where,

$$y_c = \frac{h_c}{sin\theta} = \frac{4.15}{sin50} = 5.42m$$

And from table the second moment of area can be calculated by:

$$I_{x,c} = \frac{bd^3}{12} = \frac{0.7 * 1^3}{12} = 0.05833m^4$$

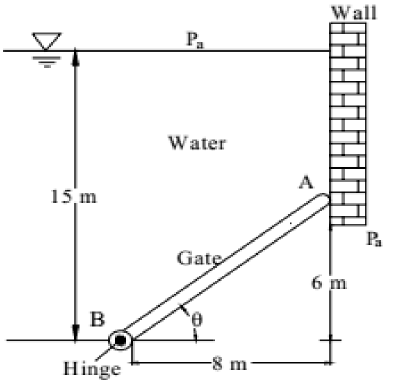
$$y_R = \frac{0.05833}{5.42 * 0.7} + 5.42 = 5.435m$$

Example 9: The gate in figure below is 5 m wide, is hinged at point B, and rests against a smooth wall at point A. Compute:

a) The force on the gate due to the water pressure.

b) The distance between the centre of gate and location of force act.

c) The reactions at hinge B.



Solution:

To start solving this, we need to find the angle first:

$$\theta = \tan^{-1}(\frac{6}{8}) = 36.87 \cong 37°$$

$$h_c = 15 - 3 = 12m$$

$$F_R = \gamma.A.h_c \rightarrow F_R = 1000 * 9.81 * 10 * 5 * 12 = 5886\ kN$$

Now, the location can be found by:

$$y_R = \frac{I_{x,c}}{y_c.A} + y_c$$

Where,

$$y_c = \frac{h_c}{sin37} = \frac{12}{sin37} = 19.94m$$

And from table the second moment of area can be calculated by:

$$I_{x,c} = \frac{bd^3}{12} = \frac{5 * 10^3}{12} = 416.67\ m^4$$

$$y_R = \frac{416.67}{19.94 * 5 * 10} + 19.94 = 20.4m$$

Note: the distance between the centre of the gate and centre of pressure can be calculated by:

$$d = y_R - y_c = 20.4 - 19.94 = 0.46m$$

Summing moments counter clockwise about B gives:

$$P * Lsin\theta = F(5 - d)$$

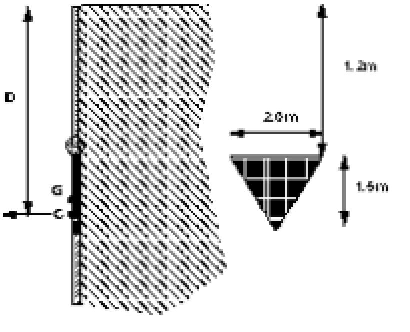
$$P * 10sin37 = 5886(5 - 0.46) \rightarrow P = 4440.3\ kN$$

With F and P known, the reactions B_x and B_z are found by summing forces on the gate.

$$\sum F_x = 0$$

$$P = Bx + \mathrm{Fsin37} \rightarrow Bx = 898\mathrm{kN}$$

Example 10: A tank holding water has a triangular gate, hinged at the top, in one wall. Find the moment at the hinge required to keep this triangular gate closed.



Solution:

The force can be calculated by:

$$h_c = 1.2 + \frac{1.5}{3} = 1.7m$$

$$A = \frac{bh}{2} = \frac{2 * 1.5}{2} = 1.5\ m^2$$

$$F_R = \gamma . A . h_c \rightarrow F_R = 1000 * 9.81 * 1.5 * 1.7 = 25\ kN$$

Now we find the distance where the force F acts:

$$y_R = \frac{I_{x,c}}{y_c . A} + y_c$$

Where,

$$y_c = h_c = 1.7m$$

And from table the second moment of area can be calculated by:

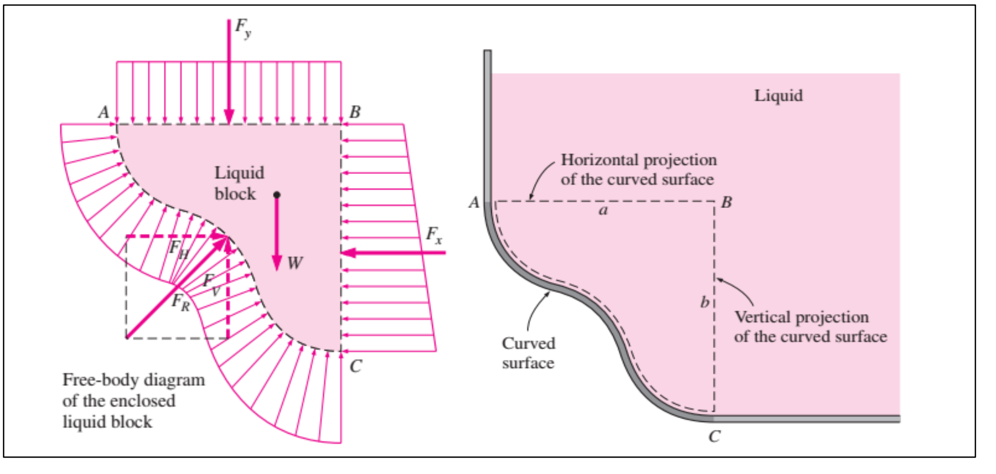
$$I_{x,c} = \frac{bh^3}{36} = \frac{2 * 1.5^3}{36} = 0.1875m^4$$

$$y_R = \frac{0.1875}{1.7 * 1.5} + 1.7 = 1.774\ m$$

The moment on the hinge from the water is: $Moment = 25 * (1.774 - 1.2) = 14.35kN$

Submerged Curved surface:

The easiest way to determine the resultant hydrostatic force FR acting on a two-dimensional curved surface is to determine the horizontal and vertical components FH and FV separately.



Horizontal force component on curved surface: $F_H = F_x$

Horizontal force component on curved surface: $F_V = F_y + W$

where the summation $F_y + W$ is a vector addition (i.e., add magnitudes if both act in the same direction and subtract if they act in opposite directions). Thus, we conclude that:

1- The horizontal component of the hydrostatic force acting on a curved surface is equal (in both magnitude and the line of action) to the hydrostatic force acting on the vertical projection of the curved surface.
2- The vertical component of the hydrostatic force acting on a curved surface is equal to the hydrostatic force acting on the horizontal projection of the curved surface, plus (minus, if acting in the opposite direction) the weight of the fluid block.

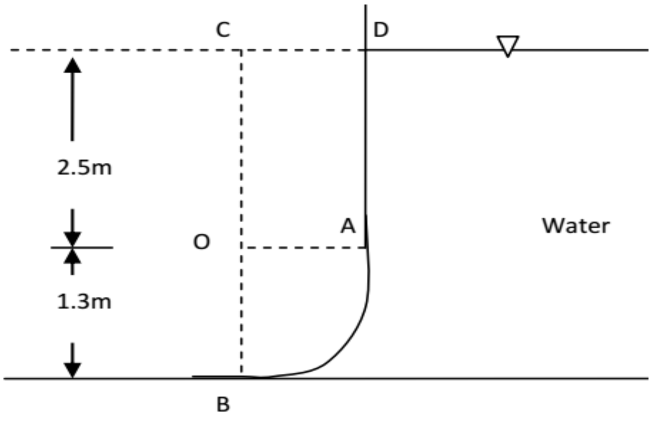
The magnitude of the resultant hydrostatic force acting on the curved surface is:

$$F_R = \sqrt{F_H + F_V}$$

And the tangent of the angle it makes with the horizontal is

$$\alpha = \frac{F_V}{F_H}$$

Example 1: The water is on the right side of the curved surface AB, which is one quarter of a circle of radius 1.3m. The tank's length is 2.1m. Find the horizontal and vertical component of the hydrostatic acting on the curved surface.



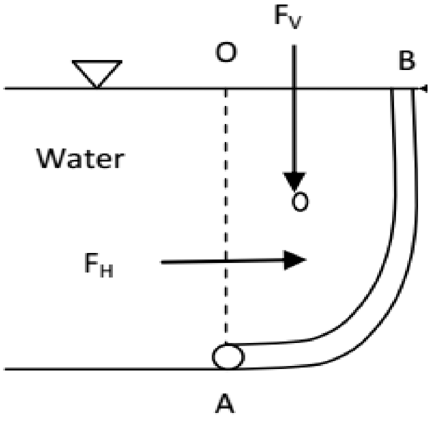
Solution:

Horizontal force, $F_H = \gamma . A . h_c \rightarrow F_H = 1000 * 9.81 * \left(2.5 + \frac{1.3}{2}\right) 1.3 * 2.1 = 84.4\ kN$

Vertical force, $F_V = \gamma . volume$

$$F_V = 1000 * 9.81 * \left(\frac{\pi}{4} * 1.3^2 * 2.1 + 1.3 * 2.1 * 2.5\right) = 94.297kN$$

Example 2: The gate AB shown is hinged at A and is in the form of quarter-circle wall of radius 12m. If the width of the gate is 30m, calculate the horizontal and vertical force.



Solution:

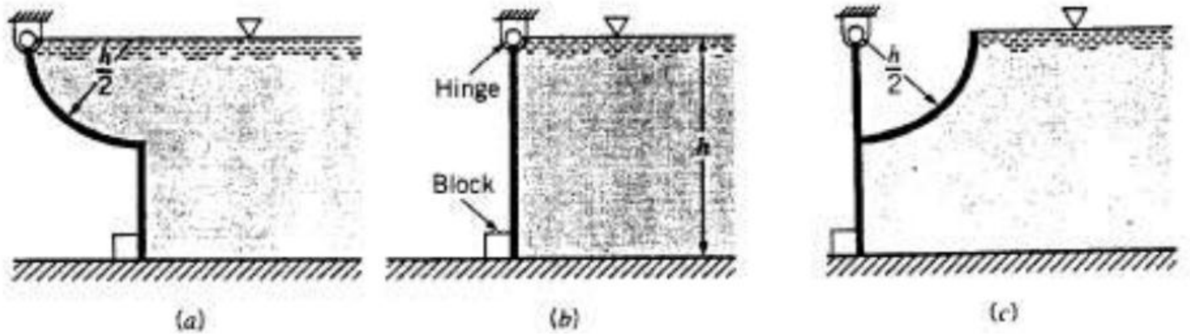
Horizontal force, $F_H = \gamma . A . h_c \rightarrow F_H = 1000 * 9.81 * \left(\frac{12}{2}\right) 12 * 30 = 21189.6\ kN$

Vertical force, $F_V = \gamma . volume$

$$F_V = 1000 * 9.81 * \left(\frac{\pi}{4} * 12^2 * 30\right) = 33284.546kN$$

Email: ammarali@wrec.uoqasim.edu.iq

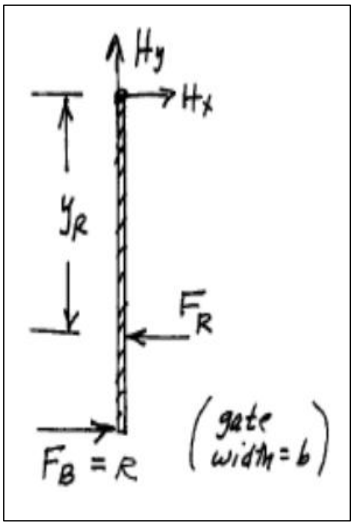
Example 3: Three gates of negligible weight are used to hold back water in the channel of width b as shown in the figure below. The force of the gate against the block for gate (b) is R. determine in term of R, the force against the blocks for the other two gates.



Solution: For gate (b)

$$F_R = \gamma_{water} * h_c * A \rightarrow F_R = 9810 * \frac{h}{2} * h * b \rightarrow F_R = 4905h^2b$$

$$y_R = \frac{I_{x,c}}{y_c.A} + y_c$$



Where,

$$y_c = h_c \rightarrow \quad y_c = \frac{h}{2}$$

And from table the second moment of area can be calculated by:

$$I_{x,c} = \frac{bd^3}{12} = \frac{bh^3}{12}$$

$$y_R = \frac{\frac{bh^3}{12}}{\frac{h}{2} * h * b} + \frac{h}{2} = \frac{h}{6} + \frac{h}{2} = \frac{4h}{6} = \frac{2h}{3}$$

Thus:

$$\sum M = 0 \rightarrow R = y_R * F_R$$

$$R = \frac{2h}{3} * 4905h^2b$$

$$R = 3270\, h^2b \quad \rightarrow eq.1$$

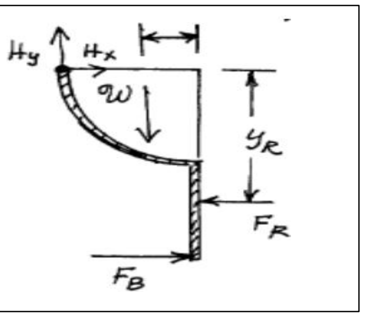
For case (a):

The weight can be calculated by: $W = 9810 * volume \rightarrow W = 9810 * \left(\frac{\pi}{4} * \left(\frac{h}{2}\right) * b\right)$

$$W = 1926.2 * h^2b$$

Email: ammarali@wrec.uoqasim.edu.iq

$$\sum M_H = 0 \rightarrow W\left(\frac{h}{2} - \frac{4h}{6\pi}\right) + F_R\left(\frac{2h}{3}\right) = F_B * h$$



This lead to:

$$F_B = 3825.9h^2b$$

With equation 1:

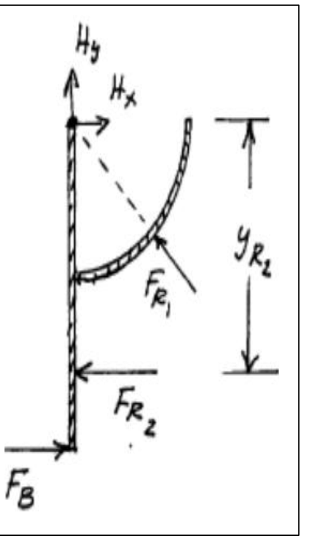
$$F_B = 1.17R$$

For case (c):

The force F_B on the curve section passes through the hinge and therefore does not contribute to the moment around H. on the bottom part of the gate:

$$F_{R2} = \gamma_{water} * h_c * A \rightarrow F_{R2} = 9810 * \frac{3h}{4} * \frac{h}{2} * b \rightarrow F_{R2} = 3678.75h^2b$$

$$y_{R2} = \frac{I_{x,c}}{y_c.A} + y_c$$



Where,

$$y_{c2} = h_{c2} \rightarrow \quad y_{c2} = \frac{3h}{4}$$

And from table the second moment of area can be calculated by:

$$I_{x,c} = \frac{bd^3}{12} = \frac{bh^3}{12}$$

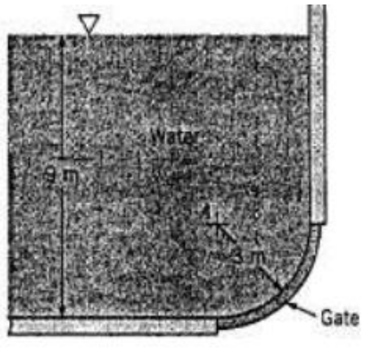
$$y_{R2} = \frac{\frac{bh^3}{12}}{\frac{3h}{4} * h * b} + \frac{3h}{4} = 0.777h$$

$$\sum M_H = 0 \rightarrow F_{R2} * 0.777h = F_B * h$$

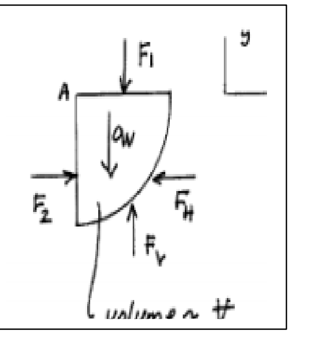
And with equation 1:

$$F_B = 0.875R$$

Example 3: A 4m long curved gate is located in the side of a reservoir containing water as shown in figure below. Determine the magnitude of the horizontal and vertical components of the force of the water on the gate.



Solution:



For equilibrium:

$$\sum F_x = 0$$

Based on the figure:

$$F_H = F_2 \rightarrow F_H = \gamma_{water} * h_c * A = 9810 * \left(6 + \frac{3}{2}\right) * 3 * 4 = 882.9\ kN$$

Similarly:

$$\sum F_y = 0$$

$$F_V = F_1 + W$$

$$F_1 = 9810 * 6 * (3 * 4) = 706.32\ kN$$

$$W = \gamma_{water} * Volume \rightarrow W = 9810 * \left(\frac{\pi}{4} * 3^2 * 4\right) = 277.4\ kN$$

Now:

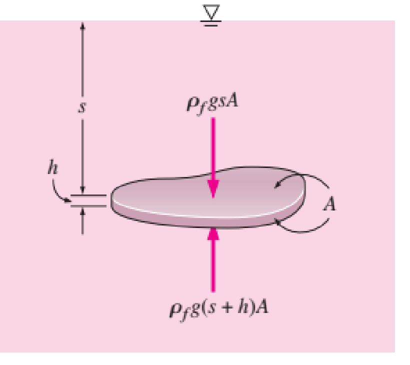
$$F_V = F_1 + W = 706.32 + 277.4 = 983.72\ kN$$

Buoyancy and Stability

It is a common experience that an object feels lighter and weighs less in a liquid than it does in air. This can be demonstrated easily by weighing a heavy object in water by a waterproof spring scale. Also, objects made of wood or other light materials float on water. These and other observations suggest that a fluid exerts an upward force on a

body immersed in it. This force that tends to lift the body is called the buoyant force and is denoted by F_B.

The buoyant force is caused by the increase of pressure in a fluid with depth. Consider, for example, a flat plate of thickness h submerged in a liquid of density ρ_f parallel to the free surface, as shown in figure below.



The area of the top (and also bottom) surface of the plate is A, and its distance to the free surface is s. The pressures at the top and bottom surfaces of the plate are $\rho_f.g.s$ and $\rho_f.g.(s+h)$, respectively. Then the hydrostatic force $F_{top} = \rho_f.g.s.A$ acts downward on the top surface, and the larger force $F_{top} = \rho_f.g.(s+h).A$ acts upward on the bottom surface of the plate. The difference between these two forces is a net upward force, which is the **buoyant force.**

$$F_{buoyant} = F_{bottom} - F_{top} = F_{top} = \rho_f.g.(s+h).A - \rho_f.g.s.A$$

$$F_{buoyant} = \rho_f.g.h.A = \rho_f.g.V, where, V = A.h\ , volume\ of\ the\ plate$$

we conclude that the buoyant force acting on the plate is equal to the weight of the liquid displaced by the plate.

Archimedes' principle:

The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume.

For floating bodies, the weight of the entire body must be equal to the buoyant force, which is the weight of the fluid whose volume is equal to the volume of the submerged portion of the floating body. That is,

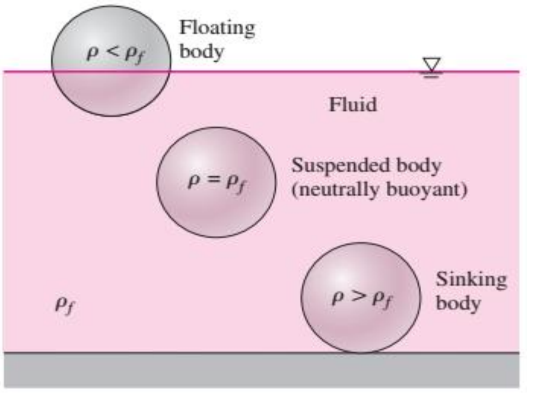
$$F_B = w \rightarrow \rho_f.g.V_{submerged} = \rho_{average,body}.g.V_{total}$$

$$\frac{V_{sub}}{V_{total}} = \frac{\rho_{average,body}}{\rho_f}$$

Notes:

1- The submerged volume fraction of a floating body is equal to the ratio of the average density of the body to the density of the fluid. Note that when the density ratio is equal to or greater than one, the floating body becomes completely submerged.

2- A body immersed in a fluid (1) remains at rest at any point in the fluid when its density is equal to the density of the fluid, (2) sinks to the bottom when its density is greater than the density of the fluid, and (3) rises to the surface of the fluid and floats when the density of the body is less than the density of the fluid.



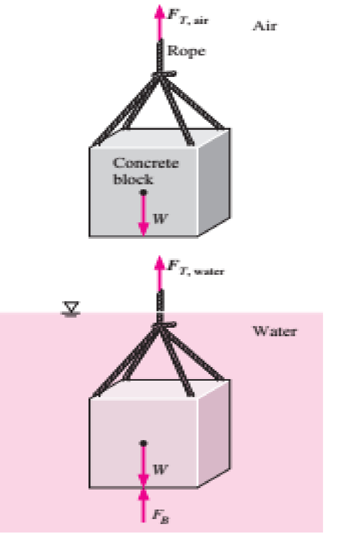
3- The buoyant force is proportional to the density of the fluid, and thus we might think that the buoyant force exerted by gases such as air is negligible. This is certainly the case in general, but there are significant exceptions. For example, the volume of a person is about 0.1 m^3, and taking the density of air to be 1.2 kg/m^3, the buoyant force exerted by air on the person is:

$$F_{buoyant} = \rho_f . g . h . A = 1.2 * 9.81 * 0.1 \cong 1.2N$$

The weight of an *80kg* person is *80 * 9.81 = 788 N*. Therefore, ignoring the buoyancy in this case results in an error in weight of just *0.15* percent, which is negligible.

Example 1: A crane is used to lower weights into the sea (density =1025 kg/m^3) for an underwater construction project as shown below. Determine the tension in the rope of the crane due to a rectangular 0.4m * 0.4m * 3m concrete block

(density = 2300 kg/m³) when it is (a) suspended in the air and (b) completely immersed in water.



Solution:

The volume of the rectangle, $V = 0.4 * 0.4 * 3 = 0.48\ m^3$

$$F_{T,air} = \rho_{concret}.g.V \rightarrow F_{T,air} = 2300 * 9.81 * 0.48 = 10.8\ kN$$

And the force when the body immersed in water is:

$$F_B = \rho_{water}.g.V \rightarrow F_B = 1025 * 9.81 * 0.48 = 4.8\ kN$$

Now the force is:

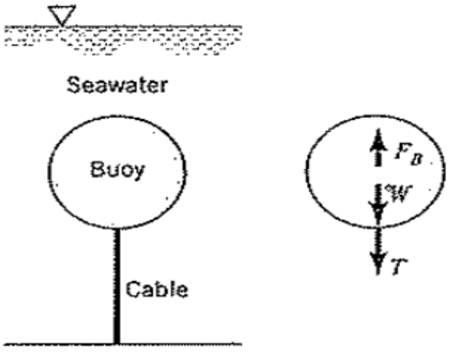
$$F_{water} = w - F_B = 10.8 - 4.8 = 6\ kN$$

Note: that the weight of the concrete block, and thus the tension of the rope, decreases by *(10.8 -6.0)/10.8 = 55%* percent in water.

Example 2: A spherical buoy has a diameter of a 1.5m, weight 8.5KN and is anchored to the seafloor with a cable as shown in figure below. Although, the buoy normally floats on the surface, at certain times the water depth increases so that the buoy is

Email: ammarali@wrec.uoqasim.edu.iq

completely immersed as illustrated. For this conditions what is the tension of the cable?



Solution:

The volume of the spherical shape, $V = \frac{\pi}{6} * D^3 = \frac{\pi}{6} * (1.5)^3 = 1.77m^3$

$$\gamma_{seawater} = 10.1 \frac{kN}{m^3}, given$$

$$F_B = \gamma_{seawater} V \rightarrow F_B = 10.1 * 1.77 = 17.877\ kN$$

The tension in the cable can be calculated by:

$$T = F_B - w = 17.877 - 8.5 = 9.377\ kN$$