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Fluid Flow in Pipes

We will be looking here at the flow of real fluid in pipes – *real* meaning a fluid that *possesses viscosity hence loses energy due to friction as fluid particles interact with one another and the pipe wall.* The difference between ideal and real fluid is that:

$$\textbf{\textit{Real fluid:}}\ \mu \neq 0$$

$$\textbf{\textit{Ideal Fluid:}}\ \mu = 0$$

The shear stress induced in a fluid flowing near a boundary is given by Newton's law of viscosity:

$$\tau\ \alpha\ \frac{du}{dy}$$

This tells us that the shear stress, τ, in a fluid is proportional to the velocity gradient - the rate of change of velocity across the fluid path. For a "Newtonian" fluid we can write:

$$\tau = \mu \frac{du}{dy}$$

Where the constant of proportionality, μ, is known as the viscosity.

A Newtonian fluid's: viscosity remains constant, no matter the amount of shear applied for a constant temperature.. These fluids have a linear relationship between viscosity and shear stress.

Examples:

- Water
- Mineral oil
- Gasoline
- Alcohol

Non-Newtonian Fluid

You can probably guess that non-Newtonian fluids are the opposite of Newtonian fluids. When shear is applied to non-Newtonian fluids, the viscosity of the fluid changes.

Recall also that flow can be classified into one of two types, **laminar** or **turbulent** flow (with a small transitional region between these two).

The non-dimensional number, the Reynolds number, Re, is used to determine which type of flow occurs:

$$Re = \frac{\rho u d}{\mu}$$

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$$\boldsymbol{Re = \frac{Inertial\ Forces}{Viscous\ Forces}}$$

For internal flow:

$$\boldsymbol{Laminar\ Flow: Re\ < 2300}$$

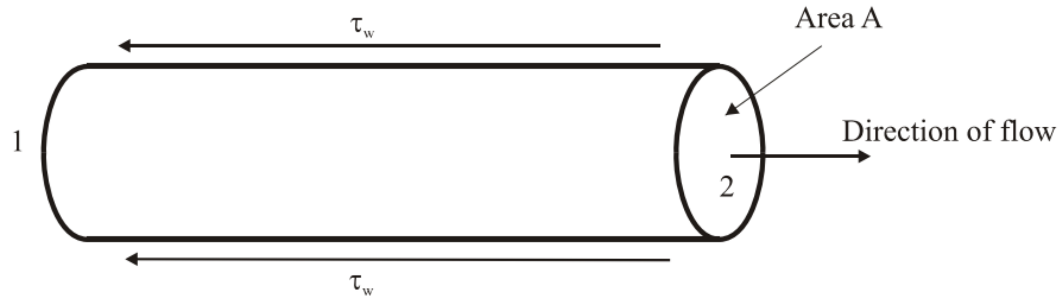
$$\boldsymbol{Transitional\ Flow: 2300\ < Re\ < 4000}$$

$$\boldsymbol{Turbulent\ Flow:\ \geq 4000}$$

It is important to determine the flow type as this governs how the amount of energy lost to friction relates to the velocity of the flow. And hence how much energy must be used to move the fluid.

Pressure loss due to friction in a pipeline

Consider a cylindrical element of incompressible fluid flowing in the pipe, as shown



The pressure at the upstream end, 1, is p, and at the downstream end, 2, the pressure has fallen by Δp to (p-Δp).

The driving force due to pressure (F = Pressure x Area) can then be written

driving force = Pressure force at 1 - pressure force at 2

$$= pA - (p - \Delta p)A = A\ \Delta p = \frac{\pi}{4} d^2 * \Delta p$$

The retarding force is that due to the shear stress by the walls

$$= shear\ stress * area\ over\ which\ it\ acts$$

$$= \tau_w * area\ of\ the\ pipe\ wall$$

$$= \ \tau_w * \pi dl$$

As the flow is in equilibrium:

driving force = retarding force

$$\frac{\pi}{4} d^2 * \Delta p = \ \tau_w * \pi dl$$

$$\Delta p = \frac{\tau_w * 4L}{d}$$

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Giving an expression for pressure loss in a pipe in terms of the pipe diameter and the shear stress at the wall on the pipe.

In general, the shear stress τ_w. is almost impossible to measure. But for laminar flow it is possible to calculate a theoretical value for a given velocity, fluid and pipe dimension. The pressure loss in a pipe with laminar flow is given by the Hagen-Poiseuille equation:

$$\Delta p = \frac{32\mu L u}{d^2}$$

Or in terms of head:

$$\boldsymbol{h_f = \frac{32\mu L u}{\rho g d^2}}$$

Where h_f is known as the head-loss due to friction.

This is known as the **Darcy-Weisbach** equation for head loss in circular pipes.

This equation is equivalent to the Hagen-Poiseuille equation for laminar flow with the exception of the empirical friction factor f introduced. It is sometimes useful to write the Darcy equation in terms of discharge Q, (using $Q = Au$).

$$u = \frac{4Q}{\pi d^2}$$

$$h_f = \frac{32 f L Q^2}{g d^5}$$

How to find the friction factor f:

1- **For laminar Flow:**

The friction factor for laminar flow can be calculated directly by using the equation below:

$$f = \frac{64}{Re}$$

2- **For Turbulent Flow:**

1- **Smooth Pipes**

Blasius, in 1913, was the first to give an accurate empirical expression for f for turbulent flow in **smooth pipes**, that is:

$$f = \frac{0.079}{Re^{0.25}}$$

2- **Rough Pipes**

Colebrook and White did a large number of experiments on commercial pipes and they also brought together some important theoretical work by von Karman and

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Prandtl. This work resulted in an equation attributed to them as the Colebrook-White equation:

$$\frac{1}{\sqrt{f}} = -4\log\left[\frac{e}{3.71\,d} + \frac{1.26}{Re * \sqrt{f}}\right]$$