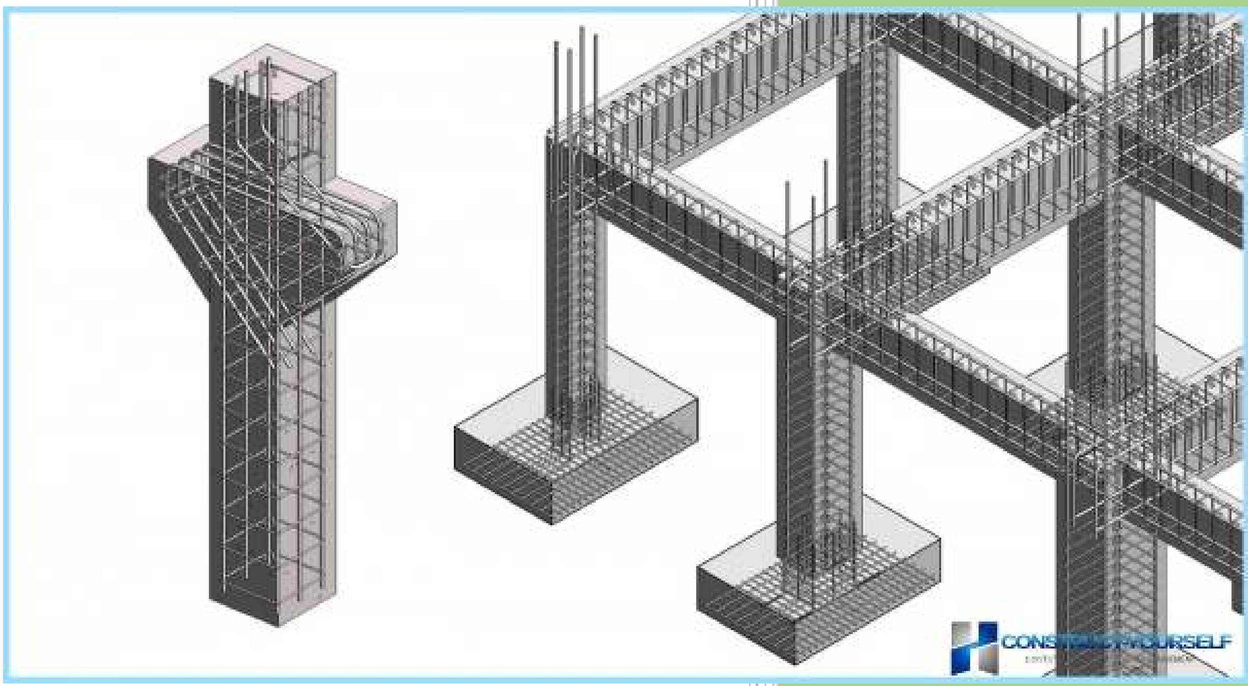
Design of Reinforced Concrete Structures I

Flexural Analysis & Design of Beam Part4

Approximate structural analysis ACI 8.8.3



Approximate structural analysis ACI 8.8.3

As an alternate to frame analysis, the following approximate moments and shears shall be permitted for design of continuous beams and one-way slabs (slabs reinforced to resist flexural stresses in only one direction), provided (a) through (e) are satisfied

(a) There are two or more spans.

(b) Spans are approximately equal, with the larger of two adjacent spans not greater than the shorter by more than 20 percent.

(c) Loads are uniformly distributed.

(d) Unfactored live load, L, does not exceed three times unfactored dead load, D.

(e) Members are prismatic.

For calculating negative moments, Ln is taken as the average of the adjacent clear span lengths.

Positive moment

End spans

Discontinuous end unrestrained .... $\frac{w_u l_n^2}{11}$

Discontinuous end integral with support .... $\frac{w_u l_n^2}{14}$

Interior spans.. $\frac{w_u l_n^2}{16}$

Negative moments at exterior face of first interior support

Two spans.. $\frac{w_u l_n^2}{9}$

More than two spans.. $\frac{w_u l_n^2}{10}$

Negative moment at other faces of interior supports...... $\frac{w_u l_n^2}{11}$

Negative moment at face of all supports for

Slabs with spans not exceeding 10 ft; and beams where ratio of sum of column stiffnesses to beam stiffness exceeds 8 at each end of the span..................... $\frac{w_u l_n^2}{12}$

Negative moment at interior face of exterior support for members built integrally with supports

Where support is spandrel beam.................................... $\frac{w_u l_n^2}{24}$

Where support is a column... $\frac{w_u l_n^2}{16}$

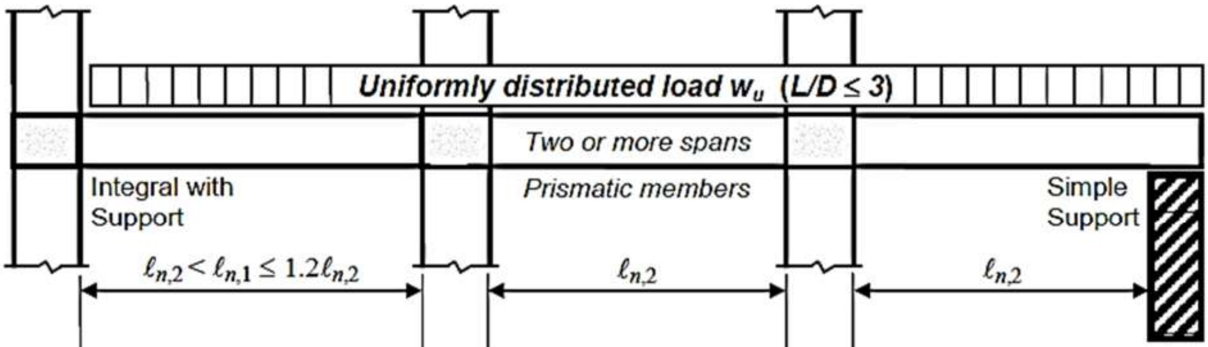
Shear in end members at face of first interior support..................... $\frac{1.15\, w_u\, l_n}{2}$

Shear at face of all other supports.. $\frac{w_u\, l_n}{2}$

DESIGN AID 1-2

Approximate Bending Moments and Shear Forces for Continuous Beams and One-way Slabs

8.3.3



	$\frac{w_u \ell_{n,1}^2}{14}$		$\frac{w_u \ell_{n,2}^2}{16}$		$\frac{w_u \ell_{n,2}^2}{11}$		**Positive Moment**
Spandrel Support	$\frac{w_u \ell_{n,1}^2}{24}$	$\frac{w_u \ell_{n,avg}^2}{10}$ *	$\frac{w_u \ell_{n,avg}^2}{11}$	$\frac{w_u \ell_{n,2}^2}{11}$	$\frac{w_u \ell_{n,2}^2}{10}$ *	0	
Column Support	$\frac{w_u \ell_{n,1}^2}{16}$						**Negative Moment**
Note A	$\frac{w_u \ell_{n,1}^2}{12}$	$\frac{w_u \ell_{n,avg}^2}{12}$	$\frac{w_u \ell_{n,avg}^2}{12}$	$\frac{w_u \ell_{n,2}^2}{12}$	$\frac{w_u \ell_{n,2}^2}{12}$ **	0	
	$\frac{w_u \ell_{n,1}}{2}$	$\frac{1.15 w_u \ell_{n,1}}{2}$	$\frac{w_u \ell_{n,2}}{2}$	$\frac{w_u \ell_{n,2}}{2}$	$\frac{1.15 w_u \ell_{n,2}}{2}$	$\frac{w_u \ell_{n,2}}{2}$	**Shear**

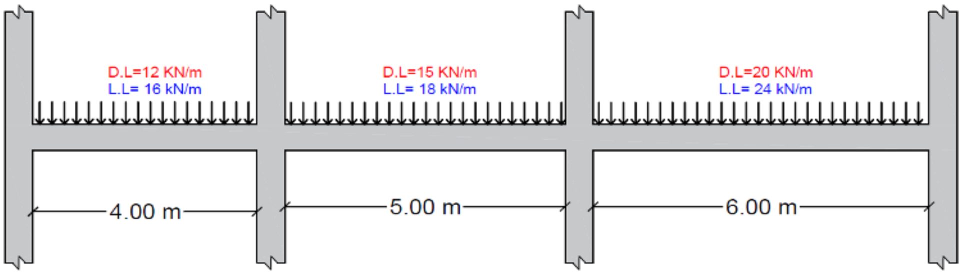
* $\frac{w_u \ell_n^2}{9}$ (2 spans)

$$\ell_{n,avg} = \frac{\ell_{n,1} + \ell_{n,2}}{2} \quad (8.3.3)$$

** $\frac{w_u \ell_{n,2}^2}{10}$ (for beams)

Note A: Applicable to slabs with spans $\le$ 10 ft and beams where the ratio of the sum of column stiffness to beam stiffness > 8 at each end of the span.

EX: A continuous beam of three span as shown in Fig. with left and right end discontinuous and integral with support. Answer the following:

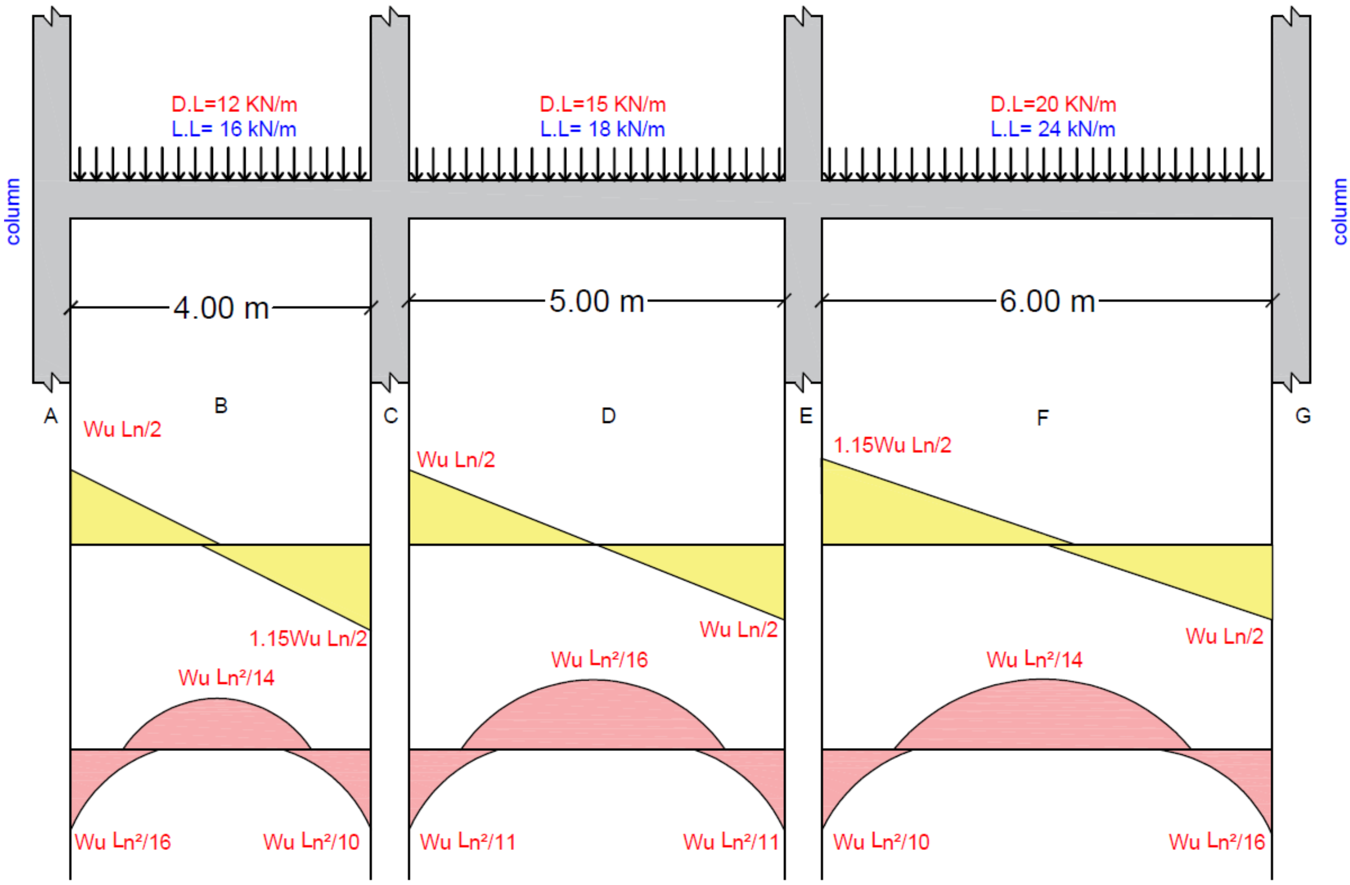


1- Draw the bending moment and shear force diagram according to ACI8.3.3.
2- Find the Max. Positive and negative moment.
3- Find the dimension?
4- Design the steel reinforcement. $\frac{f\grave{c}}{fy} = \frac{28}{350}$ MPa

Solution:

$$w_{u1} = 1.2 * 12 + 1.6 * 16 = 40 \frac{\text{KN}}{\text{m}}$$
$$w_{u2} = 1.2 * 15 + 1.6 * 18 = 46.8 \frac{\text{KN}}{\text{m}}$$
$$w_{u3} = 1.2 * 20 + 1.6 * 24 = 62.4 \frac{\text{KN}}{\text{m}}$$

2- According to ACI-code ((coefficient method)) moment can be calculated as shown below:



$$M_{u\,A} = -\frac{w_u l_n^2}{16} = -\frac{40*4^2}{16} = -40\ \mathrm{KN.m}$$

$$M_{u\,B} = +\frac{w_u l_n^2}{14} = +\frac{40*4^2}{14} = +45.71\ \mathrm{KN.m}$$

$$M_{u\,C} = -\frac{w_u l_n^2}{10} = -\frac{46.8*\left(\frac{5+4}{2}\right)^2}{10} = -94.77\ \mathrm{KN.m}$$

Note: take the maximum load in adjacent panel and average length

$$M_{u\,D} = +\frac{w_u l_n^2}{16} = +\frac{46.8*5^2}{16} = +73.13\ \mathrm{KN.m}$$

$$M_{u\,E} = -\frac{w_u l_n^2}{10} = -\frac{62.4*\left(\frac{5+6}{2}\right)^2}{10} = \mathbf{-188.76\ KN.m}$$

$$M_{u\,F} = +\frac{w_u l_n^2}{14} = +\frac{62.4*(6)^2}{14} = +160.45\ \mathrm{KN.m}$$

$$M_{u\,E} = -\frac{w_u l_n^2}{16} = -\frac{62.4*(6)^2}{16} = -140.4\ \mathrm{KN.m}$$

3- Use the maximum bending moment to find the dimension (- 188.76 KN.m)

Assume $\rho = 0.75\rho_{max.}$ $\quad where\ \rho_{max.} = 0.85\beta_1 \frac{f_c^`}{fy}\frac{0.003}{0.003+0.004} = 0.0247$

$\rho = 0.75*0.0247 = 0.0185$

$$\rho_t = 0.85\beta_1 \frac{f_c^`}{fy}\frac{0.003}{0.003+0.005} = 0.0216\ > \rho = 0.0185 \quad \rightarrow \quad \emptyset = 0.9$$

Let b=250 mm

$\emptyset M_n \geq M_u$

$$M_u = \emptyset\, \rho\, bd^2 fy \left(1 - 0.59\frac{fy}{f_c^`}\rho\right)$$

$$188.76*10^6 = 0.9*0.0185*250*d^2*350*\left(1-0.59\frac{350}{28}*0.0185\right) \rightarrow d = 399mm \approx 390\ mm$$

$$\mathrm{H = d + cover + stirrup} + \frac{d_b}{2} = 390 + 40 + 10 + 12.5 = 450\mathrm{mm}$$

Check the ACI requirement of dimension

Panel 1 one end continuous H=L/18.5 =4000/18.5=216.2mm < 450 mm .....Ok

Panel 2 both end continuous H=L/21=5000/21=238 mm < 450 mm ....Ok

Panel 3 one end continuous H=L/18.5= 6000/18.5=324.3 mm <450 mm ......Ok

4- Design the steel reinforcement for section A

$\emptyset M_n \geq M_u$

$$M_u = \emptyset\, \rho\, bd^2 fy \left(1 - 0.59\frac{fy}{f_c^`}\rho\right)$$

$$40*10^6 = 0.9*\rho*250*390^2*350*\left(1-0.59\frac{350}{28}*\rho\right)$$

$$8.833*10^{10}\rho^2 - 1.1977*10^{10} + 40*10^6 = 0 \qquad \rightarrow \rho = 0.00342$$

$$\rho_{min.} = max.of \left\{\frac{\sqrt{f_c^`}}{4\,fy} \quad \frac{1.4}{fy}\right\} = \{0.00377, \qquad 0.004\}$$

$\rho = 0.00342 < \rho_{min.=0.004}$ $\quad not\ ok \quad \rightarrow \rho = \rho_{min.=0.004}$

$AS = \rho*b*d = 0.004*250*390 = 390mm^2 \qquad \rightarrow use\ 2\emptyset16mm$