- Measures of central tendency

Those measures are looking to appreciate stationed around the data majority and these values of medium or stationed is express one number or represent all the data that group.

The most important measures of central tendency are: -

(1): The arithmetic mean.

(2): The geometric mean.

(3): The harmonic mean.

(4): The quadratic mean.

(5): The median .

(6): The mode .

(1): The arithmetic mean: It is the value resulting from dividing the sum of the observations values on its numbers and its symbol ($\bar{Y}$). The characteristics of arithmetic mean is a total deviation values to the middle arithmetic is equal zero .

- For ungrouped data is: $\bar{Y} = \Sigma (yi - \bar{y}) = 0$.

$\bar{Y} = \sum(\mathbf{y_i} - \bar{y}) = (\sum\mathbf{y_i} - \sum\bar{y}) = \sum\mathbf{y_i} - n\bar{y}) = \sum\mathbf{y_i} - n \times \sum\mathbf{y_i} / n = \sum\mathbf{y_i} - \sum\mathbf{y_i} = \mathbf{0}$ **.**

- for grouped data is : $\sum f_i(\mathbf{y_i} - \bar{y}) = 0$.

$\sum f_i(\mathbf{y_i} - \bar{y}) = \sum f_i\, \mathbf{y_i} - \bar{y} \sum f_i = \sum f_i\, \mathbf{y_i} - \sum f_i \times \sum f_i\, \mathbf{y_i} / \sum f_i = \sum f_i\, \mathbf{y_i} - \sum f_i\, \mathbf{y_i} = 0$

Also, Methods of calculating the arithmetic mean:

A. ungrouped data: if we have **(n)** replicates of $Y_{i,}$ whereas $Y_i = y_1, y_2, y_3, \ldots\ldots ..y_n$

the arithmetic mean it is $\bar{Y} = \Sigma\, y_i / n$

Example 1. The data following represents is amount of rain was falling annually (ml) on the Mosul city during the five-year period ago, 520, 350, 450 380 400 .find the average amount of rainfall .

$\bar{Y} = \Sigma yi / n$.

$\bar{Y} = (520 + 350 + 450 + 380 + 400)/5 = (2100) / 5 = 420$.

B. grouped data: if we have **($\mathbf{f_i}$)** replicates of $Y_{i,}$ whereas $Y_i = y_1, y_2, y_3, \ldots\ldots ..y_n$

the arithmetic mean it is $\bar{Y} = \Sigma fi\, y_i / fi$.

Example 2. Calculate the arithmetic mean of the lengths of cotton plants in the following table .

Categories	Repetition	Categories center	Repetition × Categories center
31 - 40	1	35.50	35.50
41-50	2	45.50	91.00
51- 60	5	55.50	277.50
61-70	15	65.50	982.50
71-80	25	75.50	1887.50
81-90	20	85.50	1710.50
91-100	12	95.50	1146.00
Total	$\Sigma f_i = 80$		$\Sigma f_i y_i = 61300$

$\bar{Y} = \Sigma f_i y_i / f_i = 6130/80 = 76.625$ cm.

This means of plant length is 76.625 cm we notes in this example (B) this is the arithmetic mean (76.625 cm) was slightly differs from the arithmetic mean in the example (A) of the same data before it classifying them and put them in frequency distribution table (76.58 cm). The difference between the two figures is due to the loss of information on the views because of its status in the totals, we suppose that the length of each plant in a particular category equal to that category center.

- The characteristics of the arithmetic mean is :

(A) The total of deviation values upon the arithmetic mean is equal zero .

unclassified data : $\sum(y_i - \bar{y}) = 0$.

$\sum(y_i - \bar{y}) = 0$.

$\sum(y_i - \bar{y}) = \sum y_i - \sum\bar{y} = \sum y_i - n\bar{y} = \sum y_i - n \times \sum y_i / n = \sum y_i - \sum y_i = 0$.

- for grouped data :

$\sum f_i(y_i - \bar{y}) = 0$.

$\Sigma f_i (y_i - \bar{y}) = \Sigma f_i y_i - \bar{y}\, \Sigma f_i = \Sigma f_i y_i - \Sigma f_i \times \Sigma f_i y_i / \Sigma f_i = \Sigma f_i y_i - \Sigma f_i y_i = 0$.

(B):The sum of deviations squares from the arithmetic mean is less than the sum of deviations squares for any value without arithmetic mean .The $\sum(y_i - \bar{y})^2$ is less than what can be.

Suppose that A is the value not arithmetic mean. prove that $\Sigma (y_i - A)^2$ is greater than the value of $\sum(\mathbf{y_i} - \bar{\mathbf{y}})^2$.

$\Sigma (y_i - A)^2 = \Sigma (y_i^2 - 2 Ay_i + A^2) = \Sigma y_i^2 - 2 A\Sigma y_i + \Sigma A^2) = \Sigma y_i^2 - 2n A\bar{y} + nA^2$.

Adding and mines $n(\bar{y})^2$ of the above produces:

$\Sigma y_i^2 - 2n A\bar{y} + nA^2 + n(\bar{y})^2 - n(\bar{y})^2 = (\Sigma y_i^2 - n(\bar{y})^2) + nA^2 - 2A\bar{y} + n(\bar{y})^2 = (\Sigma y_i^2 - n(\bar{y})^2) + n(A^2 - \bar{y})^2$

From this above that the sum squares of deviations for any value without arithmetic mean is greater than the sum squares of deviations from the arithmetic mean by $(A^2 - \bar{y})^2$ is a positive value .

Example (3):from the following values : $y_i = 7, 5, 6, 8, 9$.

$\bar{Y} = \Sigma y_i / n = (9 + 8 + 6 + 5 + 7) / 5 = 35/5 = 7.$

$\Sigma(y_i - \bar{y})^2 = (9-7)^2 + (8-7)^2 + (6-7)^2 + (5-7)^2 + (7-7)^2 = (2)^2 + (1)^2 + (-1)^2 + (-2)^2 + (0)^2 = (4) + (1) + (1) + (4) + (0) = 10.$

$\Sigma(y_i - \bar{y})^2 = 10$.

If we mines of these is the arithmetic mean and let A = 10.

$\Sigma(y_i - A)^2 = (9-10)^2 + (8-10)^2 + (6-10)^2 + (5-10)^2 + (7-10)^2 = (-1)^2 + (-2)^2 + (-4)^2 + (-5)^2 + (-3)^2 = (1) + (4) + (16) + (25) + (9) = 55.$

Of course, 55 is greater than 10. It is noted here, the difference between them is 55- 10 = 45.

It is $n(A - \bar{y})^2 = 5(10 - 7)^2 = 5(3)^2 = 5 \times (9) = 45$.

(C):When you add a fixed number (k) to the value of each of the values of observations, the arithmetic mean of the new values is equal arithmetic mean + fixed number (k) .

$X_i = y_i + k.$

$\Sigma Xi = \Sigma(y_i + k) = \Sigma yi + nk).$

$\Sigma X_i / n = \Sigma y_i / n + nk / n.$

$\bar{X} = \bar{Y} + k.$

(D):If you multiple all the value of observations by constant value (k) the arithmetic mean of the new values = arithmetic mean of the original value × fixed number .

$Z_i = k\, y_i.$

$\Sigma Z_i = k\, \Sigma y_i$

$\Sigma Z_i / n = k\, \Sigma y_i / n$

$\bar{Z} = k\, \bar{Y}.$

Example (4): From the following values:. Yi= 8, 3, 2, 12, 10 .

If you know that the valu $Z = 5y_i$ find the $\bar{Z}$.

$\bar{Y} = \Sigma yi / n = (8 + 3 + 2 + 12 + 10) / 5 = (35) / 5 = 7.$

$Z_i = 5\, y_i. = (5 \times 8, 5 \times 3, 5 \times 2, 5 \times 12, 5 \times 10)$

$Z_i = 40, 15, 10, 60, 50.$

$\bar{Z} = \Sigma Z_i / n = (40 + 15 + 10 + 60 + 50) / 5 = 175/5 = 55$.

(E):The arithmetic mean of the total two variables values = arithmetic mean for the total of medium values of two variables .

$Zi = x_i + y_i$.

$(\Sigma Z_i) / n = (\Sigma x_i) / n + (\Sigma y_i) / n$.

$\bar{Z} = \bar{X} + \bar{Y}$.

Example (5): find mathematical mean of the data in this table .

X_i	Y_i	$Z_i = X_i + Y_i$
2	5	7
4	10	14
4	8	12
8	7	15
5	10	15
$\ddot{X} = 5$	$\bar{Y} = 8$	$\acute{Z} = 13$

(E): If the value of each of the values of observations $(w_i y_i)$ has special weight is commensurate with their importance (w_i). The weighted arithmetic mean of these values is: $\bar{Y} = (\Sigma w_i y_i) / w_i$.

The following values is represent the results of students in statistics test. the weight test has importance or a certain percentage .

Example (6) : find the weighting mathematical mean of the data following :

Exam	Mark (y_i)	Task or weight or percent (w_i)	$W_i y_i$
First	70	10 %	700
Second	60	30 %	1800
Third	75	10 %	750
fourth	55	50 %	2750
Total		$\sum w_i = 100$	$\sum W_i Y_i = 6000$

$\bar{Y} = (\Sigma w_i y_i) / w_i = 6000/100 = 60$.

Example 7: Four sections of students in the first class are consists of 30, 35, 40.25 students respectively. If the results examination rate in statistic are 80,75, 60,90, respectively, what is the rate of the exam in all of these sections ?

Solution:

$\bar{Y} = (\Sigma w_i y_i) / w_i =$

$\bar{Y} = (30)(80) + (35)(75) + (40)(60) + (25)(90) / (30 + 35 + 40 + 25)$.

$\bar{Y} = 2400 + 2625 + 2400 + 2250 / 130$.

$\bar{Y} = 9675 / 130$.

$\bar{Y} = 74.42$.

(2) -The geometric mean : to find the value of the geometric mean, we use logarithms of two sides will produces the geometric mean. It is clear that the logarithm of the geometric mean of values is the arithmetic mean of the logarithms of these values. The geometric mean is symbolized by $\bar{G}$.

(A): Non-classified data: If we have n observations of $y_1, y_2, y_3 \ldots ..y_n$ the geometric mean is ($\bar{G}$).

$$\log \bar{G} = (\log Y_1 + \log Y_2 + \log Y_3 \ldots..\log Y_n) / n .$$

Example (1) : find the geometric mean of data following :

$Y_i = 3 , 5 , 8 , 3 , 7 , 2$.

Solution :

$$\log \bar{G} = (\log Y_1 + \log Y_2 + \log Y_3 \ldots..\log Y_n) / n .$$

$\log \bar{G} = (\log 3 + \log 5 + \log 8 + \log 3 + \log 7 + \log 2) / 6 =(0.4771 + 0.6990 \ldots\ldots.+ 0.3010) / 6$

$\log \bar{G} = 3.7024 /6 =$

$\log \bar{G} = 0.6171$.

$\bar{G} = 4.14$.

As about for arithmetic mean is equal to $\bar{Y} = \sum y_i/n$.

$\bar{Y} = (3 + 5 + 8 + 3 + 7 + 2)/6=$

$\bar{Y} = 28/6 =$

$\bar{Y} = 4.67$.

(B) grouped data: If our observations $y_1, y_2, y_3 \ldots ..y_n$ are represent classes centers in the frequency distribution table with replicates are $f_1, f_2, f_3, \ldots ..f_n$ geometric mean is ($\bar{G}$) .

$$\log \bar{G} = (f_1 \log Y_1 + f_2 \log Y_2 + f_3 \log Y_3 \ldots. .. f_i \log Y_i) / \Sigma f_i .$$

Example (2): find the geometric mean of distribution table :

Factions	f_i	Y_i	Log Y_i	fi logY_i
60-62	5	61	1.7782	8.8910
63-65	18	64	1.8062	32.5116
66-68	42	67	1.8261	76.6962
69-71	27	70	1.8451	49.8177
72-74	8	72	1.8633	14.9064
	100			182.8229

$$\log \bar{G} = (f_1 \log Y_1 + f_2 \log Y_2 + f_3 \log Y_3 \;....\; ..\; f_i \log Y_i) / \Sigma f_i \;.$$

$\log \bar{G} = [(5) \log 61 + (18) \log 64 + (42) \log 67 + (27) \log 70 + (8) \log 72] /100 =$

$\log \bar{G} = [\ (5)\ (1.7782) + (18)\ (1.8062) + (42)\ (1.8261) + (27)\ (1.8451) + (8)\ (1.8633)\ ] /100\ =$

$\log \bar{G} =[\ 8.8910 + 32.5116 + 76.6962 + 49.8177 + 14.9064] /100 =$

$\log \bar{G} = 182.8229 /100 =$

$\log \bar{G} = 1.828229$.

$\bar{G} = 67.3$.

$\bar{y} = (\sum f_i y_i)/f_i =[(5 \times 61) + (18 \times 64\ ) + (42 \times 67) + (27 \times 70) + (8 \times 72)]/100 =$

$\bar{y} = [(305) + (1152\ ) + (2814) + (1890) + (576)]/100 =$

$\bar{y} = (6737) /100 =$

$\bar{y} = 67.37$.