(3)-The harmonic mean: It is inverted of the arithmetic mean for the inverted of the observations .

(A): The data is ungrouped: If we have n of observations are y_1, y_2, y_3 ... ..y_n the harmonic mean is (Ḧ).

$$\bar{H} = 1 / (\sum 1/y_i) / n = n / \sum 1/y_i .$$

Example (1) : find the harmonic mean of the data following :

Y_i = 3 , 5 , 6 , 6 ,7 , 10, 12 .

Solution : $\bar{H} = n / \sum 1/y_i$.

$\bar{H}$ = 7 / [(1/3) + (1/5) + (1/6) + (1/6) +(1/7) + (1/10) + (1/12)] =

$\bar{H}$ = 7 /[(0.33) + (0.20) + (0.16) + (0.16) + (0.14) + (0.10) + (0.08)] =

$\bar{H}$ = 7/1.17 = , $\bar{H}$ = 5.98 .

Example (2):The farmer was bought a wheat seeds by price (100 dinars) from the following companies :

The first company the price of 1 ton of wheat seeds was equal (20 dinars) .

The second company the price of 1 ton of wheat seeds was equal (25 dinars) .

The third company the price of 1 ton of wheat seeds was equal (50 dinars) .

Find the average of the wheat seeds price (the harmonic mean) ?

Solution :

$\bar{H} = n /[\sum 1/y_i]$

$\bar{H}$ = 3/[(1/20) + (1/25) + (1/50)] =

$\bar{H}$ = 3/[(0.05) + (0.04) + (0.02)] = 3/0.11 =

$\bar{H}$ = 2.73 .

(B): grouped data: If our observations are y_1 , y_2 , y_3 ... ..y_n are represent classes centers in the frequency distribution table with replicates are f_1, f_2, f_3, ...f_n, the harmonic mean is (Ḧ)

$$\bar{H} = \sum f_i / \sum (f_i/y_i) .$$

Example (3) : find the harmonic mean of the distribution table in following :

Factions	f_i	y_i
60 – 62	5	61
63 - 65	18	64
66 - 68	42	67
69 - 71	27	70
72 - 74	8	73
total	100	

Solution :

$\bar{H} = \sum f_i / \sum (f_i/y_i)$.

$\bar{H} = (100)/[(5/61) + (18/64) + (42/67) + (27/70) + (8/73)] =$

$\bar{H} = (100)/[(0.08) + (0.28) + (0.63) + (0.39) + (0.11) =$

$\bar{H} = 100 / [1.4855] =$

$\bar{H} = 67.31$.

(4). The quadratic mean: It is the square root of the arithmetic mean for squares observations. Since the quadratic mean is applied very much in the physical sciences.

(A): The data is ungrouped: If we have n observations of $y_1, y_2, y_3 \ldots \ldots y_n$ the quadratic mean is (Ō).

$\bar{Q} = \sqrt{\Sigma yi^2 / n}$.

Example (1): find the quadratic mean of the data following :

$Y_i = 1, 3, 4, 5, 7.$

Solution :

$\bar{Q} = \sqrt{\Sigma yi^2 / n}$.

$\bar{Q} = \sqrt{(y1^2 + y2^2 + y3^2 + y4^2 + y5^2) / 5} =$

$\bar{Q} = \sqrt{(1^2 + 3^2 + 4^2 + 5^2 + 7^2)/5} =$

$\bar{Q} = \sqrt{(1 + 9 + 16 + 25 + 49)/5} =$

$\bar{Q} = \sqrt{100 / 5} =$

$\bar{Q} = \sqrt{20} =$

$\bar{Q} = 4.47$.

(B) The data is grouped: If our observations y_1, y_2, y_3 ... ..y_n are represent classes centers in the frequency distribution table with f_1, f_2, f_3, ...f_n the quadratic mean is ($\bar{Q}$). $\bar{Q} = \sqrt{\Sigma fiyi^2 / \Sigma fi}$ **.**

Example (2) : find the quadratic mean of the distribution table in following :

Factions	f_i	y_i	y_i^2	$f_iy_i^2$
60 – 62	5	61	3721	18605
63 - 65	18	64	4096	73728
66 - 68	42	67	4489	188538
69 - 71	27	70	4900	132300
72 - 74	8	73	5329	42632
total	100			455803

Solution :

$\bar{Q} = \sqrt{\Sigma fiyi^2 / \Sigma fi}$ **.**

$\bar{Q} = \sqrt{455803 /100} =$

$\bar{Q} = \sqrt{4558.03} =$

$\bar{Q} = 67.51$.

(5) . The median is symbol $\bar{M}$e :

If we have n of observations and arranged it in ascending or descending with are two cases :

(A): If the n is single number , the median is the value that arranged is (n + 1/2) .

(B) If n is an even number the median is the arithmetic mean of the two values which are ordered in (n / 2) , (n / 2) + 1) .

$\bar{M}$e = (Yn / 2) + [Yn / 2] + 1 / 2 .

Example (1) : find the median of the student has got marks for five tests in statistic if the scores are in below : 84 , 87 , 76 , 82 , 80 .

Ascending grades: 87, 84, 82 , 80, 76 .

The numbers is single n = 5. So median is the value that arranged =(n + 1/2) :

$\bar{M}$e = (n + 1/2) =

$\bar{M}$e = (5 + 1/2) =

$\bar{M}$e = (6/2) = 3 .

$\bar{M}e = 82$.

Example (2) : find the median of the data following : Y_i = 5 , 4 , 8 , 7 , 3 , 12 , 9 , 2 .

So must be ascending grades. The So median is inverted values that =(n/ 2) , (n / 2) + 1) .

Ascending : 12 , 9 , 8 , 7 , 5 , 4 , 3 , 2 .

$\bar{M}e = (n / 2) = (8/2) = 4$.

$\bar{M}e = (n / 2) + 1) = (8/2) +1 = 4 + 1 = 5$.

$\bar{M}e = (y_4 + y_5) / 2 = (5 + 7) / 2 = 12/2 = 6$.

- To find a median in the classified data must be the law following :

$\bar{M}\mathbf{e = L_i + [(\sum f_i /2) – F_i)/f_i]w}$

Example 1: Find the median for the frequency distribution in the following table :

Length categories	f_i	ascending accumulated		
60 - 62	5	Less than 60	0	
63 - 65	18	Less than 63	5	
66 - 68	42	Less than 66	23	
69 - 71	27	Less than 69	65	
72 - 74	8	Less than 72	92	
Total	100	Less than 74	100	

$\bar{M}e = (n + 1) /2 = (5 + 1) /2 = 6/2 = 3$.

L_i = 65.5 real minimum class of median

F_i = 23. accumulated repetitions at the beginning of category median .

f_i = 65 - 23 = 42. repetitions of category median .

w = 68.5 - 65.5 = 3. Length of category median .

$\bar{M}\mathbf{e = L_i + [(\sum f_i /2) – F_i)/f_i]\ w}$

$\bar{M}e = 65.5 + [(100 /2) – 23)/42] (3)$

$\bar{M}e = 65.5 + [(50 – 23)/42] (3)$

$\bar{M}e = 65.5 + [(27)/42] (3)$

$\overline{M}e = 65.5 + 1.928 = 67.43$.

(6).The mode :

If we have n observations of y_1, y_2, y_3 ...... y_n. The mode of these observations is most value frequent among these observations and has the symbol $\acute{M}o$.

There can be patterned one (one summit) for these observations distribution so-called single summit distribution or unimodal , or have two modes (two summits), then called a two summits or bimodal or may be more than two modes (trimodal). As he has does not have the mode of observations .

Example (1) : find the mode of the data following :

(a) : 3, **5**, 2, 6, **5**, 9, **5**, 2, 8, 6 .

(b) : 51.6, 48.7, 50.3, 49.5, 48.9 .

Solution :

(a) : **The mode is 5 .** $\overline{M}o = 5$.

(b) : **no mode found .**

-Grouped data if observations y_1, y_2, y_3 ...... y_n are representing categories centers in the frequency distribution table with replicates f_1, f_2, f_3, ... f_n the mode of these is :

$\overline{M}$**o = Li + [d1 / d1 + d2] w.**

$\overline{M}o$ = the mode for that category, which owns more iterations .

L_i = lower real limit of the class mode .

d_1 = the difference between repeating category mode and previous category .

d_2 = the difference between repeating category mode and subsequent category .

w = length of the period .

Example (2) : find the mode of the distribution table with its replicates following :

factions	f_i
60- 62	5
63 – 65	18
66 - 68	42
69 -71	27
72 - 74	8
Total	100

The Category (66-68) have high duplicates (42) is called category mode .

$\bar{M}o = Li + [d1 / d1 + d2]\ w$.

$Li = 65.5$ lower real limit of the class mode .

$d1 = 42 - 18 = 24$. The difference between repeating category mode and previous category .

$d2 = 42 - 27 = 15$. the difference between repeating category mode and subsequent category .

$w = 3$. The length of the period .

$\bar{M}o = L_i + [d_1/d_1 + d_2]w$.

$\bar{M}o = 65.5 + [24/24 + 15](3) =$

$\bar{M}o = 65.5 + [0.556](3) =$

$\bar{M}o = 65.5 + 1.674 =$

$\bar{M}o = 67.174$.